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Analysis of magnetoelastic interaction of rectangular ferromagnetic plates with nonlinear magnetization

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Abstract

In this paper, a theoretical model is developed to describe a magnetic force arisen from an interaction between the nonlinear magnetization of structures made of soft ferromagnetic materials and an applied magnetic field in which the structures are placed. The fundamental equations of ferromagnetic structures in a magnetic field involve twofold nonlinearity since the interaction between the magnetic field and the deformation of the structure leads to the equations nonlinear besides the nonlinear magnetization. Magnetoelastic bending and buckling of a simply supported rectangular plate, as an example, are investigated by these fundamental equations. A numerical programme combined the 3D FEM with the iterative method is suggested to quantitatively simulate the magnetoelastic behavior of the plate. It is found that the characteristics of magnetoelastic bending and instability for the plates having nonlinear magnetization are distinctive from those of the plates having linear magnetization.

Keywords: Rectangular ferromagnetic plates; Nonlinear magnetization; Nonlinear interaction; Magnetoelastic bending and instability; Numerical analysis

1. Introduction

Ferromagnetic materials such as ferrite, carbon steel, silicon steel and alloy steel etc., as a kind of common one, are widely used in engineering. When these materials are placed in an applied magnetic field, they may be magnetized. Following their magnetization, the structures made of ferromagnetic materials are deformed by the action of magnetic forces arisen from the interaction of the magnetization and the magnetic field. Further, the magnetic fields, both in and out of the structures may be changed with the deformation of the structures. This kind of deformations usually include the deformations out of the midplane of the structures, which are called the magnetoelastic bending, and the ones in the midplane of structures, which are mainly arisen from magnetostriction. For some special materials, such as grand

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magnetostriction materials (i.e. GMM), more attention should be paid to the change of dimensions due to magnetostriction. However, for some normal ferromagnetic materials, more attention has been paid to magnetoelastic bending deformations. It has been found that for magnetoelastic bending deformations, the magnetoelastic interaction between the magnetic field and the mechanical behavior is nonlinearly coupled even if the linear deformation and the linear magnetization are considered to the structures in applied magnetic fields (Zhou et al., 1995).

Moon and Pao (1968) experimentally discovered the magnetoelastic buckling/instability of a cantilevered ferromagnetic beam-plate in a uniform transverse magnetic field, which was called a negative magnetic-stiffness phenomenon. On the contrary, Takagi et al. (1993) experimentally found that the natural frequency of a cantilevered ferromagnetic beam-plate with low susceptibility in an in-plane magnetic field increases with an increasing magnetic field intensity, which was called a positive magnetic-stiffness phenomenon. In order to characterize these experimental phenomena, several theoretical models and numerical programme were proposed in publications for the ferromagnetic materials having linear magnetization (e.g., Moon and Pao, 1968; Pao and Yeh, 1973; Miya et al., 1980; Eringen, 1989; Zhou et al., 1995; Zhou and Zheng, 1997). However, Zhou and Miya (1998) found that only the Zhou–Zheng's model (1997) can commonly describe the two distinct experiments.

It should be noted that the models listed above are for the linear magnetized materials. With the application of ferromagnetic structures in a strong magnetic field (higher than 1 T), such as the structure of first wall in a fusion reactor, the magnetic field generated in the structures may be up to the region of saturation of the constitutive relation of magnetic fields. In this case, the nonlinearity of magnetization varying with magnetic field has to be considered in a theoretical model or a numerical programme. Miyata and Miya (1988) developed a numerical programme to analyze the magnetoelastic interaction of ferromagnetic structures with nonlinear magnetization based on the Moon–Pao's model (1968). However, the effect of deformation of the structure on the magnetic force exerted on the structure was not considered in their numerical programme.

In this paper, we try to develop a theoretical model for ferromagnetic plates having nonlinear magnetization and nonlinear interaction between the deformation of the plate and magnetic fields by generalizing the Zhou–Zheng's model (1997) which can commonly describe the negative and the positive magnetic-stiffness phenomena. A numerical programme of 3D FEM with an iterative method is established to analyze magnetoelastic bending and buckling/instability of a simply supported rectangular plate in a transverse magnetic field or in the field with a tiny incident angle. The numerical results display some interesting characteristics of the plate having nonlinear magnetization, which are very different from those having linear magnetization.

2. Theoretical model

Let us consider a soft ferromagnetic rectangular thin plate with length a, width b and thickness h in a stationary applied magnetic field $\mathbf{B}_0$ (shown in Fig. 1). For simplicity, the assumptions of homogeneous and isotropic, but nonlinear magnetization ferromagnetic plate without electric field, charge distribution and current are adopted. The magnetic constitutive relation between the magnetic field vector $\mathbf{H}$ and magnetization $\mathbf{M}$ or magnetic induction $\mathbf{B}$ can be written as follows

$$\mathbf{M}^+ = \chi_m \mathbf{H}^+ \quad \text{in } \Omega^+ \tag{1a}$$

$$\mathbf{M}^- = \mathbf{0} \quad \text{in } \Omega^- \tag{1b}$$

or

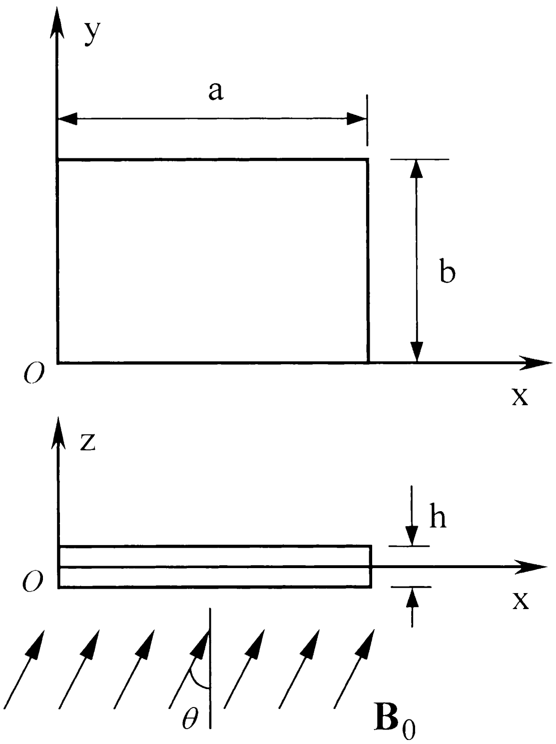

Fig. 1. The rectangular ferromagnetic plate in an applied magnetic field.

$$\mathbf{B}^+ = \mu_m \mathbf{H}^+ \quad \text{in } \Omega^+ \tag{2a}$$

$$\mathbf{B}^- = \mu_0 \mathbf{H}^- \quad \text{in } \Omega^- \tag{2b}$$

in which the superscripts "+" and "−" are respectively used to distinguish between inside and outside of the plate, for example, Ω^+ represents the region inside of the plate and Ω^- represents the outside one; μ_0 is the magnetic permeability of vacuum. χ_m and μ_m are the magnetic susceptibility and the magnetic permeability of ferromagnetic materials, respectively. When χ_m and μ_m are respectively taken as a constant, magnetic constitutive relation, i.e. Eq. (1a) or Eq. (2a) becomes linear one, which states that the magnetization of the materials is linear one. However, when the magnetic field in the structure is closed to its saturation field, as a result of the saturation effect of the magnetization for ferromagnetic medium, χ_m and μ_m are the function of $H^+ = |\mathbf{H}^+|$ respectively, namely

$$\chi_m = \chi_m(H^+), \quad \mu_m = \mu_m(H^+) \quad \text{in } \Omega^+ \tag{3}$$

Having substituted Eq. (3) into Eq. (1a) or Eq. (2a), one can find that the magnetic constitutive relation Eq. (1a) or Eq. (2a) becomes nonlinear one, which states the magnetization of the materials is nonlinear.

When there are no mechanical forces exerted on the thin plate with small deflection, the total energy of magnetoelastic system can be expressed as

$$\begin{aligned}\Pi\{\phi, w\} &= \Pi_{em}\{\phi, w\} + \Pi_{me}\{\phi, w\} \\ &= \int_{\Omega^+(w)} \left(\int_0^{H^+} B^+ \, \mathrm{d}H^+ \right) \mathrm{d}v + \frac{1}{2} \int_{\Omega^-(w)} \mu_0 (\nabla\phi^-)^2 \, \mathrm{d}v + \int_{S_0} \mathbf{n} \cdot \mathbf{B}_0 \phi^- \, \mathrm{d}s \\ &\quad + \frac{1}{2} \int_{S^*} D \left\{ (\overline{\nabla}^2 w)^2 - 2(1-v) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} \mathrm{d}x \, \mathrm{d}y \end{aligned} \tag{4}$$

where $\Pi_{em}\{\phi, w\}$ is magnetic energy of the magnetoelastic system; $\Pi_{me}\{\phi, w\}$ is strain energy of the deformed ferromagnetic plate; $\phi = \phi(x, y, z)$ is a magnetic scalar potential function and we have $\mathbf{H} = -\nabla\phi$; $w = w(x, y)$ is the deflection of the plate; $\mathbf{n}$ and S^* denote the normal direction and the region of mid-plane of the deformed plate, respectively; $\nabla = (\partial/\partial x)\mathbf{i} + (\partial/\partial y)\mathbf{j} + (\partial/\partial z)\mathbf{k}$ is a 3D gradient operator; $\overline{\nabla}^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$ is a Laplace operator in the mid-plane of the plate; $D = Eh^3/12(1 - v^2)$ represents the flexural rigidity of the plate; E is Young's modulus, and v is Poisson's ratio.

Having considered the arbitrariness and independence of $\delta\phi$ and δw, one can easily get, from $\delta\Pi\{\phi, w\} = 0$, all fundamental equations respectively for magnetic fields and ferromagnetic plate as follows:

Governing equations for magnetic fields

$$\nabla(\mu_m \nabla\phi^+) = 0 \quad \text{in } \Omega^+(w) \tag{5a}$$

$$\nabla^2\phi^- = 0 \quad \text{in } \Omega^-(w) \tag{5b}$$

Jumping conditions for magnetic fields

$$\phi^+ = \phi^-, \quad \mu_m \frac{\partial\phi^+}{\partial n} = \mu_0 \frac{\partial\phi^-}{\partial n} \quad \text{on } S \tag{6}$$

Boundary condition for magnetic field

$$\nabla\phi^- = -\frac{1}{\mu_0}\mathbf{B}_0 \quad \text{at } \infty \text{ or } S_0 \tag{7}$$

Deflection equation for ferromagnetic plate

$$D\overline{\nabla}^2\overline{\nabla}^2 w(x, y) = q_z^{em}(x, y) \tag{8}$$

Here, S is a boundary surface of the deformed plate; S_0 is an enclosed surface involved the magnetic field and the plate but far away from the plate; $q_z^{em}(x, y)$ is an equivalent magnetic force exerted on the plate and can be expressed as

$$q_z^{em}(x, y) = \left[\frac{\mu_m^2}{2\mu_0}(H_n^+)^2 + \frac{\mu_0}{2}(H_\tau^+)^2 - \int_0^{H^+} B^+ \, \mathrm{d}H^+ \right]_{z=-h/2}^{z=h/2} \tag{9}$$

where H_n^+ and H_τ^+ are the normal and tangential components of $\mathbf{H}^+$ on the surface S of the ferromagnetic plate. It should be noted that the integration term in Eq. (9) is dependent on the constitutive relation of the materials between the magnetic induction $\mathbf{B}$ and the magnetic field vector $\mathbf{H}$. Usually, it is not easy to get an analytic expression for this integration term. However, when the constitutive relation B–H or B–M is a linear one, that is, the linear magnetization of the materials is taken as

$$\mathbf{B}^+ = \mu_0\mu_r\mathbf{H}^+ \quad \text{in } \Omega^+ \tag{10}$$

where μ_r is relative magnetic permeability, the integration term can be expressed explicitly as follows

$$q_z^{em}(x,y) = \frac{\mu_0\mu_r(\mu_r - 1)}{2}\{[\mathbf{H}_n^+(x,y,h/2)]^2 - [\mathbf{H}_n^+(x,y,-h/2)]^2\} - \frac{\mu_0(\mu_r - 1)}{2}\{[\mathbf{H}_\tau^+(x,y,h/2)]^2 - [\mathbf{H}_\tau^+(x,y,-h/2)]^2\} \tag{11}$$

which is consistent with the one got by Zhou and Zheng (1997). For a transverse magnetic field, we have $H_n^+ \gg H_\tau^+$, and the magnetic force subjected on the plate can be simplified as

$$q_z^{em}(x,y) = \frac{\mu_0\mu_r(\mu_r - 1)}{2}\{[\mathbf{H}_n^+(x,y,h/2)]^2 - [\mathbf{H}_n^+(x,y,-h/2)]^2\} \tag{12}$$

which is just as same as the model suggested by Zhou and Zheng (1997) to predict the Moon–Pao's experiment (1968) not only in quality, but also in quantity as well. For a magnetic field in the plane of the plate, we have $H_\tau^+ \gg H_n^+$, so the magnetic force subjected on the plate becomes

$$q_z^{em}(x,y) = -\frac{\mu_0(\mu_r - 1)}{2}\{[\mathbf{H}_\tau^+(x,y,h/2)]^2 - [\mathbf{H}_\tau^+(x,y,-h/2)]^2\} \tag{13}$$

which is just the model derived by Zhou and Miya (1998) to predict the Takagi et al.'s experiment (1993) of the natural frequency of a ferromagnetic beam plate increasing with the applied magnetic field. Consequently, the Zhou–Zheng's model (1997), which is valid only for the materials having linear magnetization, is a special case of the model derived in this paper, and is involved in Eq. (9).

It should be noted that Eq. (8) is a nonlinear partial differential equation on unknown deflection $w(x,y)$ even for the plate having linear magnetization. It is because that the interaction between the magnetic field and the deformation is considered in the model suggested in this paper, which leads the magnetic scalar potential function $\phi(x,y,z)$ or the magnetic field vector $\mathbf{H}$ in the plate is related to the deflection of the plate. Once the nonlinear magnetization of the materials is considered, Eq. (5a) becomes a nonlinear one, too, by Eq. (3). Therefore, the fundamental Eqs. (5a)–(9) display twofold nonlinearity. It is usually not easy to obtain an analytical solution to this kind of coupled and nonlinear partial differential equations.

3. Solution programme

In order to know the deformation of the plate by solving the Eqs. (5a)–(9), a numerical programme combined the finite element method with an iterative method is suggested in this paper. The finite element methods are proposed for solving the magnetic fields inside and outside of the deformed plate as well as for solving the deflection of the plate. The iterative methods are used to respectively deal with the twofold nonlinearity which one is from the nonlinear magnetization and the other is from the interaction between the deformation and the magnetic field.

3.1. FEM analysis for magnetic field

Here, we will numerically analyze the magnetic field distribution in regions Ω^+ and Ω^- which are influenced, respectively, by the magnetization of the ferromagnetic plate and the deformation of the plate. For a given deflection $w_0(x,y)$ of the ferromagnetic plate, the magnetic field distribution (solution of Eq. (5a)–(7)) minimizes the magnetic energy functional $\Pi_{em}\{\phi, w\}$. Having discretized the 3D magnetic field regions, that is Ω^+ and Ω^- into several small elements and introduced shape function $[\mathbf{N}(x,y,z)]_e$ to these elements, one can easily get, from $\delta_\phi\Pi_{em}\{\phi, w\} = 0$, the algebraic equation of magnetic scalar potential of the nodes for each element

$$[\mathbf{K}]_e[\mathbf{\Phi}]_e = [\mathbf{P}]_e \tag{14}$$

in which $[\mathbf{\Phi}]_e$ is a column matrix which consists of the value of ϕ on each node of the element e. The magnetic stiffness matrix $[\mathbf{K}]_e$ and the column matrix $[\mathbf{P}]_e$ of inhomogeneous term for element e are respectively written by

$$[\mathbf{K}]_e = \begin{cases} \int_{\Omega_e} \mu_0 [\nabla\mathbf{N}]_e^{\mathrm{T}} [\nabla\mathbf{N}]_e \,\mathrm{d}v & \Omega_e \in \Omega^-(w) \\ \int_{\Omega_e} \mu_m [\nabla\mathbf{N}]_e^{\mathrm{T}} [\nabla\mathbf{N}]_e \,\mathrm{d}v & \Omega_e \in \Omega^+(w) \end{cases} \tag{15}$$

$$[\mathbf{P}]_e = \begin{cases} -\int_{S_{e_0}} \mu_0 \mathbf{n} \cdot \mathbf{B}_0 [\mathbf{N}]_e^{\mathrm{T}} \,\mathrm{d}s & S_{e_0} \in S_0 \\ \mathbf{0} & S_{e_0} \notin S_0 \end{cases} \tag{16}$$

Having assembled the element magnetic stiffness matrices of all elements, we can write the global algebraic equation for the magnetic field in the form

$$[\mathbf{K}][\mathbf{\Phi}] = [\mathbf{P}] \tag{17}$$

where $[\mathbf{\Phi}]$ is a column matrix of the unknown values of the magnetic potential at all nodes; $[\mathbf{K}]$ is the global magnetic stiffness matrix; $[\mathbf{P}]$ is the matrix related to the applied magnetic field on S_0. Since Ω^+ denotes the region inside of the deformed plate and the magnetic permeability μ_m is a function of magnetic field, it is obvious that

$$[\mathbf{K}] = [\mathbf{K}([\mathbf{W}], [\mathbf{\Phi}])] \tag{18}$$

in which $[\mathbf{W}]$ denotes the column matrix of the deflections at the nodes of the ferromagnetic plate. It should be noted that Eq. (17) shows not only the physical nonlinearity from the nonlinear magnetization, but also the coupled nonlinearity from the interaction between the magnetic field and the deformation of the plate.

3.2. FEM analysis for plate

Taking the rectangular areas of those 3D hexahedron elements in the mid-plane of plates as the plate elements of the finite element method, we can reduce the differential Eq. (8) with the corresponding boundary conditions into a matrix equation

$$[\mathbf{A}][\mathbf{W}] = [\mathbf{Q}] \tag{19}$$

Here, $[\mathbf{A}]$ is the stiffness matrix for deflection of the plate; $[\mathbf{Q}]$ is a column matrix of the equivalent magnetic force which is related to $q_z^{em}(x, y)$ in Eq. (9). Thus, $[\mathbf{Q}]$ should be the function of the magnetic field, and the deformation field because of the magnetic field distribution depending on the deformation of the plate, that is

$$[\mathbf{Q}] = [\mathbf{Q}([\mathbf{\Phi}], [\mathbf{W}])] \tag{20}$$

3.3. Iterative arithmetic for nonlinearity

Since there exists twofold nonlinearity in the fundamental equations of the magnetoelastic system which we investigated here, we first solve the nonlinearity from the magnetization of the ferromagnetic plate by the Newton–Raphson Method. For a given configuration (as $[\mathbf{W}^*]$) of a small deflection of the plate, we can write the magnetic scalar potential as follows

$$[\mathbf{\Phi}_{m+1}] = [\mathbf{\Phi}_m] - [\mathbf{J}_m]^{-1} \{ [\mathbf{K}_m([\mathbf{W}^*], [\mathbf{\Phi}_m])][\mathbf{\Phi}_m] - [\mathbf{P}] \} \tag{21}$$

$$[\mathbf{J}_m] = \frac{\partial}{\partial[\mathbf{\Phi}]} \{ [\mathbf{K}([\mathbf{W}^*], [\mathbf{\Phi}])[\mathbf{\Phi}] - [\mathbf{P}]] \}_{[\mathbf{\Phi}_m]} \tag{22}$$

in which [**J**] is the Jacobian matrix; the superscript '−1' represents the inverse of a matrix; and m denotes the number of iterations. The Jacobian matrix of an arbitrary element $[\mathbf{J}]_e$ can be expressed as

$$[\mathbf{J}]_e = \int_{\Omega_e} \mu^* [\nabla \mathbf{N}]^{\mathrm{T}} [\nabla \mathbf{N}] \, \mathrm{d}v + \int_{\Omega_e} 2 \frac{\partial \mu^*}{\partial (\mathbf{H})^2} [\nabla \mathbf{N}]^{\mathrm{T}} [\mathbf{T}]_e [\mathbf{T}]_e^{\mathrm{T}} [\nabla \mathbf{N}] \, \mathrm{d}v \tag{23}$$

where

$$[\mathbf{T}]_e = [\nabla \mathbf{N}]_e [\mathbf{\Phi}]_e, \quad \mu^* = \begin{cases} \mu_m & \Omega_e \in \Omega^+ \\ \mu_0 & \Omega_e \in \Omega^- \end{cases} \tag{24}$$

An iterative procedure is used to get the magnetic scalar potential $[\mathbf{\Phi}_m]$. First, we assume an initial value $[\mathbf{\Phi}_0]$ (for example, choose the magnetic scalar potential value for the case of linear magnetization) for a configuration of small deflection of the plate $[\mathbf{W}^*]$, and obtain the corresponding magnetic stiffness matrix $[\mathbf{K}_0([\mathbf{W}^*], [\mathbf{\Phi}_0])]$ from Eq. (15). Then, we calculate the Jacobian matrix $[\mathbf{J}_m]$ from Eqs. (21)–(24) and the magnetic potential $[\mathbf{\Phi}_1]$ from Eq. (21). It is obvious that $[\mathbf{\Phi}_1]$ is usually different from $[\mathbf{\Phi}_0]$. So, we replace the initial iterative potential $[\mathbf{\Phi}_0]$ with the iterated potential $[\mathbf{\Phi}_1]$, and then repeat aforementioned programme to get a new iterated potential $[\mathbf{\Phi}_m]$ $(m = 2, 3, \ldots)$ until the condition

$$\|[\mathbf{\Phi}_{m+1}] - [\mathbf{\Phi}_m]\| < \epsilon_1 \tag{25}$$

is satisfied, here $0 < \epsilon_1 \ll 1$ is a pregiven precision. So, we can finally obtain $[\mathbf{\Phi}_m]$.

For nonlinear equation (8), an iterative method is also employed. Once a small deflection $w_0(x, y)$ or $[\mathbf{W}_0]$ of plate for an applied magnetic field $\mathbf{B}_0$ is assumed, one can obtain the magnetic field distribution for the nonlinear magnetization by Eqs. (17) and (21), and calculate the equivalent magnetic force by Eq. (9). With the aid of solving Eq. (19), one can get the new deflection $[\mathbf{W}_1]$ of plate. Having substituted $[\mathbf{W}_1]$ for the initial value $[\mathbf{W}_0]$ and repeating the solving procedure about $[\mathbf{W}_1]$ until the following precision condition

$$\|[\mathbf{W}_{n+1}] - [\mathbf{W}_n]\| < \epsilon_2 \tag{26}$$

is satisfied. Here, n is the number of the iterations; $0 < \epsilon_2 \ll 1$ is a prescribed tolerance. By above programme, one can obtain the magnetoelastic bending solution $[\mathbf{W}_n]$ for the rectangular ferromagnetic plate under an applied magnetic field $\mathbf{B}_0$.

It is obvious that the convergency of above process is sensitive to the initial deflection $w_0(x, y)$ or $[\mathbf{W}_0]$ chosen in the iterative procedure. In order to fast the convergency, one makes the magnitude of the applied magnetic field $[\mathbf{B}_0]$ increase step-by-step, and take the iterative initial deflection $w_0(x, y)$ or $[\mathbf{W}_0]$ for a given magnetic field $\mathbf{B}_0 + \Delta\mathbf{B}_0$ as the one for the last applied magnetic field $\mathbf{B}_0$. The validity is confirmed in our numerical simulation.

4. Numerical results and discussion

In order to show the effect of nonlinear magnetization on magnetoelastic behavior of ferromagnetic plates, we pay our attention to a simply supported rectangular plate because its critical magnetic field intensity of magnetoelastic buckling is usually beyond the saturation magnetic field. Some geometric and materials parameters of the plate are taken as $E = 1.2 \times 10^5$ Mpa, $v = 0.3$, $a = 0.1$ m, $b = 0.1$ m and $h = 0.0015$ m. The linear magnetization relation is taken as $B = \mu_0 \mu_r H$, and the nonlinear one is taken as $B = H/(\alpha_1/\mu_0 + \alpha_2 H) + \mu_0 H$, (Mayergoyz, 1986), shown in Fig. 2, in which $\mu_r = 1000$, $\alpha_1 = 0.001$ and $\alpha_2 = 0.15$.

First of all, magnetoelastic buckling of the plate with nonlinear magnetization in a transverse magnetic field, whose direction is normal to the mid-plane of the plate, is simulated by the theoretical model and the numerical programme suggested in this paper. The critical magnetic field values $B^*_{\mathrm{cr}} (= B_{\mathrm{cr}}/\sqrt{\mu_0 E} \times 10^4)$ for

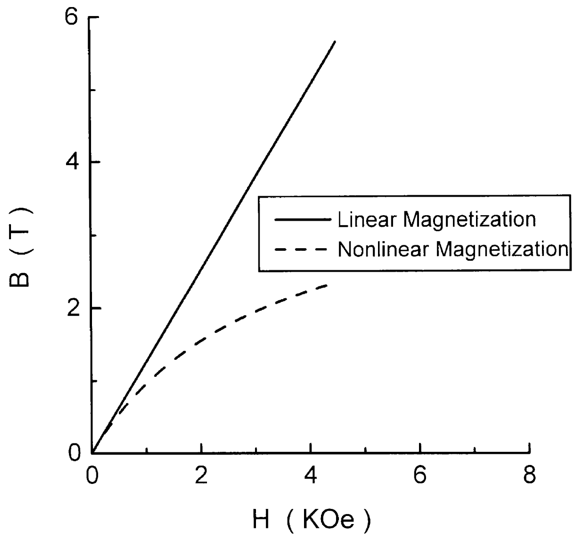

Fig. 2. The magnetization *B–H* curves for soft ferromagnetic material.

the magnetoelastic buckling of the plates with a given width b and different ratios of length a to thickness h are obtained. The corresponding logistic curves are shown in Fig. 3. It is found that the predicted critical magnetic field values exhibit a $-3/2$ power dependence on the geometric parameter a/h, which is the same with the case of linear magnetization, but the critical values for the plate having nonlinear magnetization are little bit higher than those for the plate having linear magnetization. So, when the structure is in the case of nonlinear magnetization, its critical magnetic field values of magnetoelastic buckling are higher than those of the structures having linear magnetization.

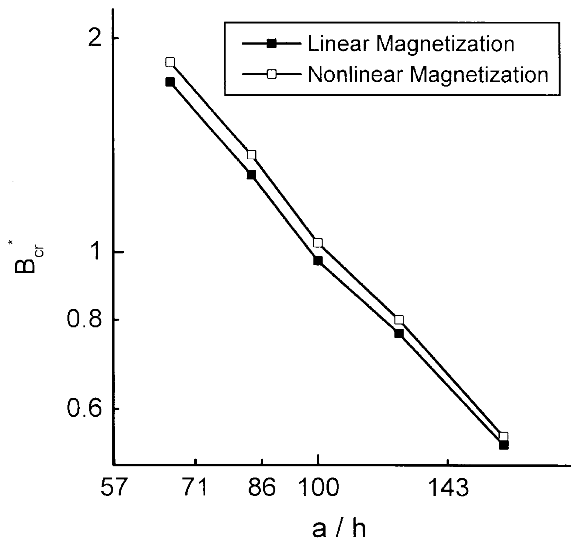

Fig. 3. The logistic curves for the critical magnetic field value B^*_{cr} vs. the geometric parameter a/h.

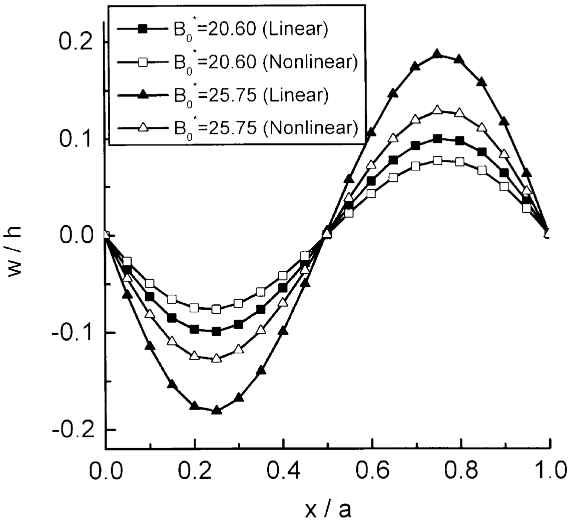

Fig. 4. The deflection curves of the simply supported plate in an oblique field ($y = b/2$, $\theta = 2°$).

Next, we calculate the magnetoelastic bending deformation of the plate having nonlinear magnetization in an oblique magnetic field whose incident angle is denoted as θ. Its deformed shape displays one whole wave-typed in the x-direction and one half wave-typed in the y-direction, respectively. The shape of the deformed plate is symmetry in the x-direction, and is antisymmetry in the y-direction, which is similar as that with linear magnetization. However, there exist differences in the values of the deflection between the nonlinear magnetization plate and the linear one. The deflection curves of the plate at $y = b/2$ are plotted in Fig. 4 for two kinds of magnetization and a given incident angle $\theta = 2°$. It is found, from Fig. 4, that the deflection values of the plate having nonlinear magnetization are smaller than those with linear magnetization. The deflection differences increase with an increasing magnetic field intensity (see Fig. 4). The stronger the magnetic field intensity, the bigger the difference.

In addition, the deflection curves of the plate at $y = b/2$ are plotted in Fig. 5 for the case of nonlinear magnetization as well as several different values of the angle θ. It states that the increment in deflection of the plate having nonlinear magnetization is different from one with linear magnetization. For the plate with linear magnetization, the deflection increases with increasing the incident angle for a given magnetic field intensity. However, for the plate having nonlinear magnetization, from Fig. 5, one can find that the deflection increases only when the incident angle is small, such as $\theta < 4°$. When $\theta > 4°$, the deflection will decrease with an increasing the incident angle. In order to clearly show this characteristic of the plate having nonlinear magnetization, the maximum deflections of the plate with linear and nonlinear magnetization are respectively shown in Fig. 6 for several values of the incident angle θ. Fig. 6 displays that the maximum deflection of the plate with linear magnetization increases fast when the magnetic field intensity B_0 is close to a critical one, which means that the shape of deformed plate may be unstable or the plate may buckle. But this situation will only take place for some small incident angles of the magnetic field when the plate is nonlinearly magnetized. For the bigger incident angle, such as $\theta = 4°$, the maximum deflection of the plate having nonlinear magnetization increases very slowly, which means the plate may not buckle. Besides that, for a given magnetic field intensity B_0, the bigger the incident angle, the smaller the maximum deflection, which is contrary to the case for the plate with linear magnetization.

Finally, the maximum deflection of the plate vs. the incident angle is plotted in Fig. 7. It states that the difference of the deformation between linear magnetization plate and nonlinear one becomes more and

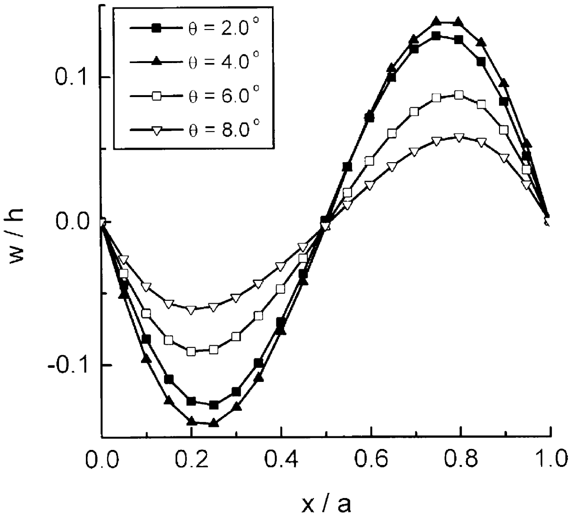

Fig. 5. The deflection curves of the simply supported plate with nonlinear magnetization ($y = b/2$, $B_0^* = 20.60$).

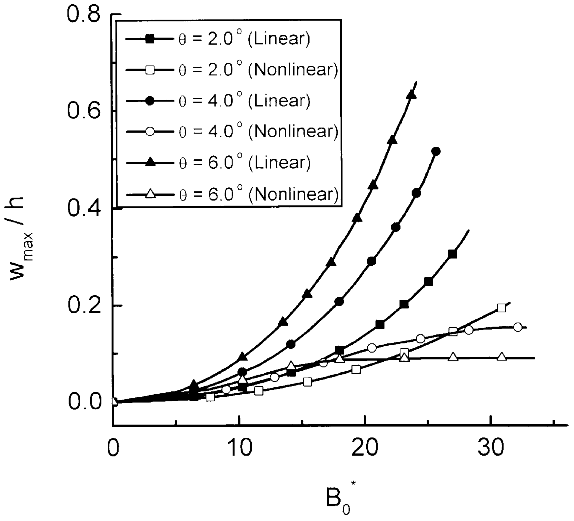

Fig. 6. The maximum deflection of the simply supported plate vs. the applied magnetic field intensity.

more obvious with an increasing magnetic field intensity B_0. These obvious differences in mechanical behavior of the plate between the linear and nonlinear magnetization arise from the nonlinear *B–H* magnetization curve. When the applied magnetic field is close to the saturation field of the soft ferromagnetic materials, of Fig. 2, the inner magnetic field of magnetized materials increases slowly with an increasing applied magnetic field intensity B_0, and tends to a steady state. Consequently, the equivalent magnetic force exerted on the plate increases slowly, even contrarily decreases due to the saturation of magnetization, which results in the deflection of the plate having nonlinear magnetization increasing slowly, or decreasing.

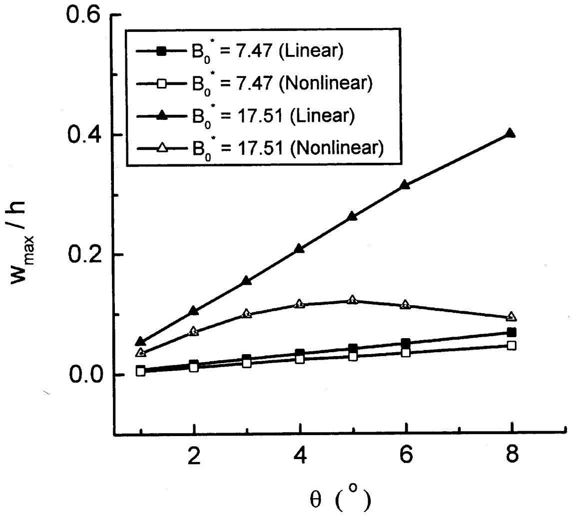

Fig. 7. The maximum deflection of the simply supported plate vs. the incident angle.

5. Conclusions

Magnetoelastic bending and buckling of a rectangular ferromagnetic plate having nonlinear magnetization are simulated in this paper. An expression of the equivalent magnetic force is suggested to describe the interaction between the magnetic field and the plate having nonlinear magnetization. The fundamental boundary-value equations exhibit twofold nonlinearity. One is from the nonlinear magnetization of the materials, and the other is from the coupled influence between the deformation of the plate and the distributions of the magnetic fields both inside and outside of the plate. The finite element method is employed for the plate and the magnetic field, and the iterative method is used for the nonlinearity. The numerical results show that the critical magnetic field values for magnetoelastic buckling of the plate having nonlinear magnetization are lower than those of the plate with linear magnetization. And the deflection vs. the incident angle θ for the nonlinear magnetization plate was found to be changed with increasing the incident angle, which is not similar to the case of linear magnetization. Since the effect of the nonlinear magnetization on the deformation of the plate becomes obvious with increasing the magnetic field intensity, more attention should be paid to the influence of the nonlinear magnetization of materials, especially when the magnetic field, in which some ferromagnetic structures are placed, is beyond the saturation one of the materials.

Acknowledgements

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References

Eringen, A.C., 1989. Theory of electromagnetic elastic plates. Int. J. Engng. Sci. 27, 363–375.
Mayergoyz, I.M., 1986. Mathematical models of hysteresis. IEEE Trans. Magn. MAG-22, 603–606.

Miya, K., Takagi, T., Ando, Y., 1980. Finite-element analysis of magnetoelastic buckling of ferromagnetic beam-plate. ASME J. Appl. Mech. 47, 377–382.

Miyata, K., Miya, K., 1988. Magnetic field and stress analysis of saturated stell. IEEE Trans. Magn. 24, 230–233.

Moon, F.C., Pao, Y.H., 1968. Magnetoelastic buckling of a thin plate. ASME J. Appl. Mech. 35, 53–58.

Pao, Y.H., Yeh, C.S., 1973. A linear theory for soft ferromagnetic elastic solids. Int. J. Engng. Sci. 11, 415–436.

Takagi, T., Tani, J., Matsubara, Y., Mogi, I., 1993. Electromagneto-mechanics coupling effects for non-ferromagnetic and ferromagnetic structures. In: Miya, K. (Ed.), Proceedings of the Second International Workshop on Electromagnetic Forces and Related Effects on Blankets and Other Structures Surrounding Fission Plasma Torus, September, Japan, pp. 81–90.

Zhou, Y.H., Miya, K., 1998. A theoretical prediction of natural frequency of a ferromagnetic beam plate with low susceptibility in an in-plane magnetic field. ASME J. Appl. Mech. 65, 121–126.

Zhou, Y.H., Zheng, X.J., Miya, K., 1995. Magnetoelastic bending and snapping of ferromagnetic plates in oblique magnetic fields. Fusion Engng. Des. 30, 325–337.

Zhou, Y.H., Zheng, X.J., 1997. A general expression of magnetic force for soft ferromagnetic plates in complex magnetic fields. Int. J. Engng. Sci. 35, 1405–1407.